Simple Model

Below is a simple theory model that sketches out some predictions to test for in the empirical section:

Assume there are two types of workers: young (y) and old (o). Both have a uniform distribution of ability $\delta \in [0,1]$. There are $\overline{N}_y$ and $\overline{N}_o$ potential young and old workers, respectively. There is one type of firm in the economy, that can choose to search for either young or old workers. The firms create vacancies until the expected zero profit condition binds.

Assume that old workers *must* be out of work sick that is equivalent to productivity s_o . Equivalently, young workers *must* be out of work sick that is equivalent to productivity s_y . Assume for simplicity that $s_y = 0$.

Utility of old and young workers are defined thusly:

$$U_o = P_o(Wage_o + \alpha s_o) - K$$

$$U_y = P_y\left(Wage_y\right) - K$$

Where α is a measure of utility gained from being home to nurse a potential illness (compared to having to come to work), and K is a search cost that must be paid to seek to match with a vacancy. Assume again, for simplicity, that $K = 1$. The probability of finding a vacancy for old and young workers is defined by:

$$P_o = \frac{x_o}{N_o}$$

$$P_y = \frac{x_y}{N_y}$$

The number of matches x_k where *k=y* or *o*, is a Cobb-Douglas on the interior:

$$x_k = \ min\,\{\gamma J_k^{\theta} N_k^{\theta}, J_k, N_k\}$$

The γ is a normalizing constant, and $\theta \in (0,1)$. For simplicity, we assume $\theta = \frac{1}{2}$, which is consistent with a CRS matching function where the number of vacancies (J_k)and number of searchers(N_k) equally contribute to the probability of matching with each other.

Wage is defined by a Rubenstein bargaining game with firms, such that for the worker takes the share of the productivity generated. The bargaining power of the worker is $\beta, \beta \in (0,1)$. Therefore, the wage for old and young workers, respectively, is:

$$Wage_o = \beta(\delta_i - s_o)$$

$$Wage_y = \ \beta(\delta_i)$$

Clearly, for two workers with *identical* abilities, under the model with no mandated paid-sick-leave, younger workers are paid a higher wage than older workers, since they are more productive, due to less days out sick.

Then it is trivial to show that the marginal old (young) worker to search is:

$$\delta_o^* = \left[\frac{1}{P_o} + (\beta - \alpha)s_o\right]\frac{1}{\beta}$$

$$\delta_y^* = \left[\frac{1}{P_y}\right]\frac{1}{\beta}$$

That is, old and young workers find it worthwhile to search for a job as long as their ability is above the cut off ability value defined above. Note that for similar values of probability of matching from the worker side, $\delta_o^* < \delta_y^*$, since we expect $(\beta - \alpha) > 0$ (as is the nature of involuntary sick days) .Since ability is distributed uniform, the number of old and young searchers is defined by:

$$(1 - \delta_o^*) \cdot \bar{N}_o = N_o$$

$$\left(1 - \delta_y^*\right) \cdot \bar{N}_y = N_y$$

Firms search for old and young workers according to the following expected zero-profit condition:

$$q_o(1 - \beta)(E(\delta|o) - s_o) - C = 0$$

$$q_y(1 - \beta)E(\delta|y) - C = 0$$

Where $q_k$is the probability of the firm successfully matching with a worker in $k = y \text{ or } o$. Note that because of the uniform distribution assumption, we can plug in for $E(\delta|o)$= $\frac{1-\delta_o^*}{2}$ and $E(\delta|y)$= $\frac{1-\delta_y^*}{2}$.

Solving for the fraction of searching workers over the number of vacancies: $\frac{N_k}{J_k}$ and simplifying, we find that:

$$\frac{N_k}{J_k} = \frac{\sigma^2 \pm (\sigma^4 - 4\sigma^2\omega - 2\omega)^{1/2}}{2}$$

Where $\sigma = (\beta - (3\beta - \alpha)s_k)\gamma$ and $\omega = \frac{2\beta C}{(1-\beta)}$. There are two potential solutions, where the ratio in question can *increase or decrease* depending on whether we use

$$\frac{N_k}{J_k} = \frac{\sigma^2 - (\sigma^4 - 4\sigma^2\omega - 2\omega)^{1/2}}{2}$$

Or

$$\frac{N_k}{J_k} = \frac{\sigma^2 + (\sigma^4 - 4\sigma^2\omega - 2\omega)^{1/2}}{2}$$

The easiest way to see this is to note that as you increase the number of sick leaves you must take, your ability draw must also increase to make it worth your while to search for a position. Therefore, while the firm loses productivity from a sicker worker because he/she is absent more often, he/she is also more productive from the start (or in expectation, any worker will be more productive).

Therefore, paradoxically, the sicker worker in expectation will be more productive, and in some cases, will face higher or lower probability of unemployment, depending on the equilibrium, which is determined initially by the ratio of searching workers to vacancies.

Now, if a paid-sick-leave mandate is introduced, the worker and firm decisions change in the following manner:

$$U_o = P_o(Wage_o + \alpha s_o) - K$$

$$U_y = P_y\left(Wage_y + \alpha_m s_m\right) - K$$

Here, s_m is the mandated amount of paid-sick-leave. For young workers, α_m represents the value of "forced" sick days. They do not need to stay home desperately to nurse an illness, but they probably do get some amount of enjoyment out of it. Importantly, we assume that α_m is quite small, certainly smaller than β: the point being that if they would trade in the paid-sick-leave for extra pay if they could. We assume that $s_m < s_o$. Note that the older worker utility function is identical. However, younger worker utility changes, since the worker may as well take the "free" vacation days offered.

The wages change in the following manner:

$$Wage_o = \beta(\delta_i - s_o) + \beta s_m$$

$$Wage_y = \beta(\delta_i - s_m) + \beta s_m = \beta\delta_i$$

Note that the last term is the paid-sick-leave that is "returned" to the worker. Because our model is short-run, we assume sticky prices (wages), and firms cannot adjust the productivity measure to account for the paid-sick-leave.

Then, proceeding as before, we find:

$$\delta_o^* = \left[\frac{1}{P_o} + (\beta - \alpha)s_o\right]\frac{1}{\beta} - s_m$$

$$\delta_y^* = \left[\frac{1}{P_y} - \alpha_m s_m\right]\frac{1}{\beta}$$

And the firm problem changes to:

$$q_o[(1-\beta)(E(\delta|o)-s_o)-\beta\, s_m]-C=0$$

$$q_y[(1-\beta)(E(\delta|y)-s_m)-\beta s_m]-C=0$$

We do not solve for the complete model here, but two points demonstrate the insight from our empirical section. First, unsurprisingly, because of the $-s_m$ term, δ_o^* must decline, compared to the no mandate regime. This means that *more* older workers are now searching (we are hand-waving a bit, since P_o must also adjust). Younger workers increase search in the market as well, but the amount of change depends critically on α_m. If this is very small, as we assume above, the change in the number of young workers is most likely very small, compared to the regime with no mandate.

From the firm side, we see that recruitment for both young and old workers is dampened due to the extra burden of the paid-sick-leave: $-\beta s_m$. However, older workers are hit particularly hard here, because the $E(\delta|o)$ term is expected to decline precipitously. This is because a greater number of older workers who are less productive enter the market, driving down the expected random productivity draw from older workers. Then, to make up for this, q_o must increase substantially. The only way this can be accomplished is for firms to severely pull back on the number of vacancies for older workers: J_o.

We predict that the labor force participation increase for young workers will be zero to a very small positive number. Employment change for young workers is also predicted to be a small negative number. Firms face higher cost, but increased LFPR of young workers will result in a higher probability of match from the firm side.

A paid-sick-leave mandate is expected to result in more older workers searching and higher unemployment for older workers. Note here that we do not allow firms to selectively choose higher ability older workers. If this type of selection is allowed, the impact for poor older workers would be even more severe.

Note that this is assuming that employment costs do not change as a result of mandate sick leave. If costs increase with increased absenteeism such that $\frac{\partial C}{\partial s}>0$ (for example, due to higher monitoring costs at the time of hiring), then the reduction in the number of vacancies for older workers would be even greater, leading to even larger negative employment effects.